Punching in Slabs with Shear Reinforcements: a Tensile Failure

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Summary

Following recent results demonstrating that punching failure in reinforced concrete structures is influenced by the tensile behavior of concrete, a model to compute the punching strength was developed based on the integration of the concrete tensile stresses. This model is extended here to treat in a unified way different types of shear reinforcement in function of their bond properties. The contribution of studs made with plain bars and anchorage is expressed in terms of the fracture energy of concrete and reflects a size-effect inversely proportional to the stud length. The contribution of stirrups with high-bond bars is equal to the yield strength of stirrups as long as the transmission length is available. The developed analytical expressions are successfully compared to experimental results.

1 Introduction

The punching failure of a reinforced concrete slab is due to concentrated load (introduced for example by a column) and occurs when a conical plug of concrete-delimitated by the punching crack-perforates the slab just above the column, followed by a sudden decrease of the load carrying capacity. In order to increase the failure load and to reduce the sudden decrease of the load carrying capacity, slabs are reinforced with shear reinforcements such as studs, stirrups, bent-bars or bolts. The computation of the failure load in such slabs should differentiate the following failure mechanisms (illustrated in fig. for a slab with studs).

1. A failure mechanism for which the punching crack is located in between the column face and the first row of shear reinforcement; the computation of the corresponding failure load should consider the interaction between the punching and the flexural strength in terms of the punching crack inclination as proposed by Menétrey [5].
2. A failure mechanism for which the punching crack is initiated outside the last row of shear reinforcement; the punching strength is computed similarly to the one of a normal reinforced concrete slab except that the radius of punching crack initiation is equal to the radius of the last row of shear reinforcements; the influence of this radius on the size-effect law should be considered as proposed by Menétrey [4].
3. A failure mechanism for which the punching crack crosses the shear reinforcement.

The computation of the punching load for the third failure mechanism is specially addressed here. This issue has to be considered because models proposed by most of the standards to estimate the contribution of the shear reinforcement to the punching strength are ambiguous. The main reason is illustrated by considering a slab reinforced with shear reinforcement which fails in one section, the punching crack. For this slab, according to most of the standards, the punching strength is computed in two different sections: (1) a vertical control section for the concrete strength (determined as the integral of the shear stress) and (2) another section which cannot be vertical for the strength of the shear reinforcements. Second reason to reconsider the treatment of the shear reinforcement is that the value of it's contribution is not uniformly accepted (varies from 50% to 100% of the yield strength of the shear reinforcement).

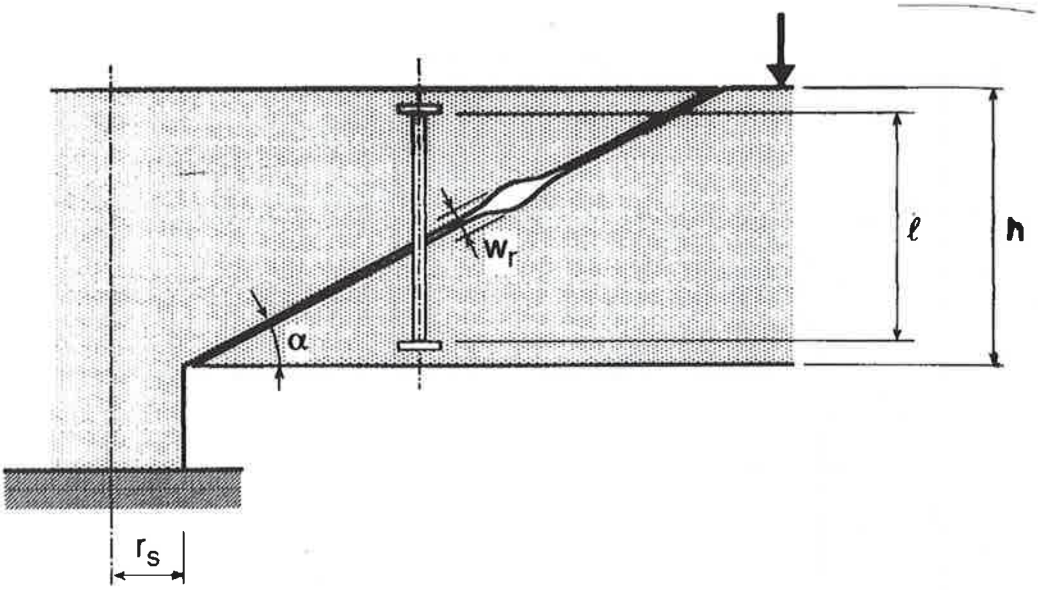

Fig. 3 Punching crack formation in slab reinforced with studs

The failure mechanism for which the punching crack crosses the studs is initiated by micro-cracks. Due to micro-cracking the slab thickness increases, resulting in the loading of the studs. Consequently, the studs sustain a so-called displacement control loading for which the displacement corresponds to the summation of the micro-cracks opening. This micro-cracks formation is followed by the crack coalescence and the crack propagation which crosses the studs. Between the crack coalescence and the crack propagation, they are insignificant increase of load (Menétrey et al. [7]) and no sudden change of the slab thickness (Regan [8]). Therefore, the elongation of the studs at failure is expressed in terms of the crack rupture opening ω_r as illustrated in fig. 3 by projection of the crack opening perpendicularly to the axis of the studs so that $\Delta l = \omega_r \cos \alpha$ (α = inclination of the punching crack). The studs deformation is consequently $\varepsilon_s = \Delta l / l = \omega_r \cos \alpha / l$ and the maximum force is expressed using equ. 2 as:

$$V_s = A_s E_s \frac{\omega_r \cos\alpha}{l} = A_s E_s \frac{5G_f \cos\alpha}{f_t l} < F_y \tag{4}$$

This force is limited by the yield strength of the studs expressed as $F_y = f_s A_s$ up to the stud length $l_0 = \omega_r \cos \alpha \, E_s/f_s$ and for higher studs length, the contribution to the punching strength is reduced, reflecting a size-effect inversely proportional to the studs length. It can also be observed that the maximum force in the studs is proportional to the fracture energy and inversely proportional to the tensile strength of concrete

The same argument can be applied to determine the contribution of stirrups with high-bond bars. During micro-cracks formation, the stirrups are crossed by various micro-cracks so that the tensile force inside the stirrups is distributed along the micro-cracked zone and is transmitted beyond this micro-crack zone to concrete by bond stresses. The tensile force in the stirrups is transmitted by bond stresses to concrete over the transmission length (defined as the length over which slip between steel and concrete occurs CEB-FIP Model Code [1]). If this length is available, the yield strength of the stirrups $F_y = f_s A_s$ is reached. If this length is not available, the carrying force in the stirrup is function of the anchorage of the stirrup's extremity. Consequently, the anchorage of the stirrups has to be carefully set, specially in the top face of slabs (positive bending) and for shorts stirrups.

4 Comparison with experimental results

The analytical contribution of the shear reinforcements is controlled with the few experimental results available for which the punching crack crosses the shear reinforcements. The studs contribution is controlled for two groups of tests summarized in tab. 1 and the stirrups contribution is controlled for one test.

The two slabs tested by Van der Voet et al. [10] are considered (abbreviated VdV): slab MV1 without shear reinforcement (punching load 375 kN) and slab MV6 (punching load 502 kN) for which the punching crack crosses 36 studs. It can be observed that the total yield strength of the studs (217 kN) does not correspond to the increase of the punching load (ΔV_S = 127 kN). However, by applying equation 4 with a crack opening ω_r=0.15 mm (assumed) and a punching crack inclination α=30° the maximum force in the studs V_s=133°kN is very close to experimental increase of the punching load.

Similarly, the analytical computation of the studs contribution is controlled for two series of slabs tested by Tolf [9]. First, slabs S1.1-2 without shear reinforcement (punching load 205 kN) and S1.1-2s with shear reinforcements (punching load 260 kN) and second, slabs S2.3-4 without shear reinforcement (punching load 466 kN) and S2.3-4s with shear reinforcements (punching load 553 kN) are considered with a slab thickness of 120 mm respectively 240~mm. For both series of tests: (1) shear reinforcements are made with plain bars so that eq. \ref{eq:s:v_sti} developed for studs is applicable, (2) only the first row of studs is crossed by the punching crack, (3) the crack opening is computed by applying the method given by the CEB-FIP Model Code [1] with a mean compressive strength of 31 MPa and a maximal aggregate size of 16 mm for slabs S1 and 32 mm for slabs S2. As shown in tab. 1 the analytical results are close to the experimental ones. Furthermore, by comparing the two test series, it can be observed that a simple strength criterion F_y cannot be employed.

slab	1	studs	f_s	F_y	ω_r	V_s	ΔV_{exp}	$V_s/\Delta V_{exp}$
	mm	Nb ∅ mm	MPa	kN	mm	kN	kN	%
VdV;MV6	137	36 ∅ 4.9	325	217	0.15	133	127	104
Tolf; S1.1s, S1.2s	105	16 ∅ 5	620	194	0.11	59	55	107
Tolf; S2.3s, S2.4s	210	8 ∅ 10	660	414	0.18	97	87	111

Tab. 1 Comparison of analytical and experimental studs contributions

The influence of high-bond stirrups is controlled with the thick slab (730 mm) tested by Kinnunen et al. [3]. The reference slab S1 reaches a punching load of 4915 kN and slab S2 reinforced with high-bond stirrups reaches a punching load of 8320~kN so that the increase of punching strength due to the stirrups obtained experimentally is ΔV_{exp}=3400 kN. Only the first row of stirrups is crossed by the punching crack (72 stirrups of diameter 12 mm and yield strength f_s=428 MPa). The stirrups contribution reaches the yield strength V_s=F_y=3480 kN, which is verified to be very close to the experimental contribution (102%).

Conclusion

The punching failure of reinforced concrete structures is influenced by the tensile behavior of concrete. The model developed to compute the punching strength by integrating the tensile stresses around the punching crack is well adapted to include the contribution of the shear reinforcements.

The bond properties of shear reinforcements allow to distinguished different types of contribution to the punching strength. For studs made with plain bars, it is shown that the strength contribution is expressed in terms of the fracture energy and is inversely proportional to the studs length, reflecting a size-effect. By comparing with experimental results, it is shown that a simple yield strength criterion cannot be used for studs. For stirrups made with high-bond, the tensile force is transmitted to concrete with bond stress along the transmission length and the stirrup contribution reaches the yield strength as long as the transmission length is available or the stirrups are well anchored.

For practical applications it results that the developed model can be efficiently used for design purposes and that shear reinforcements with high-bond are more efficient than ones with plain bars, especially for thick slabs.

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